FTD-TT- 65-649

AD 622452

TT65-64177

TRANSLATION

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By

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English pages: 7

SOURCE: Leningrad. Politekhnicheskiy Institut. Trudy. (Russian), No. 230, 1964, pp. 7-12.

S/2563-064-000-230

TT5002003

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EFFECT OF UNEQUILIBRIZED DISSOCIATION ON FLOW AROUND BLUNT-NOSED BODIES

by

Yu. P. Lun'kin and M. P. Shtengel'

In this study we consider the effect of unequilibrized dissociation of a diatomic gas on the position of the shock wave and the profile of the gas-dynamic parameters along the zero line of the flow of a blunt-nosed body.

1. Basic Equations

Let us consider the flow around an axisymmetrical body with a spherical head part by a diatomic disassociating gas. Since the time of excitation of the revolving and oscillating degrees of freedom is much less than the time of the dissociation relaxation, therefore in agreement with the method of quasi-equilibrized zones [1] it is possible to consider that the inner degrees of freedom in the process of dissociation find themselves in a state of equilibrium.

$$\frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta v_r}{r} + \frac{1}{\rho r}\frac{\partial p}{\partial \theta} = 0; \qquad (1)$$

$$\frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} + v_r\frac{\partial v_r}{\partial r} - \frac{v_\theta^2}{r} + \frac{1}{\rho}\frac{\partial p}{\partial r} = 0; \qquad (2)$$

$$r\frac{\partial \rho v_r}{\partial r} + 2\rho v_r + \frac{\partial \rho v_\theta}{\partial \theta} + \rho v_\theta \operatorname{ctg}\theta = 0. \qquad (3)$$

$$v_r\frac{\partial \alpha}{\partial r} + \frac{v_\theta}{r}\frac{\partial \alpha}{\partial \theta} = C\rho\left[(1-\alpha)e^{-\frac{D}{RT}} - \frac{\rho}{\rho_\infty}\alpha^2\right] \equiv \omega(p,\ T). \qquad (4)$$

Here v_θ and v_r are components of the speed;

ρ, p, T, α are density, pressure, temperature and degree of dissociation;

D is the energy of dissociation;

C, ρ_{oo} are constants which characterize the respective speed of dissociation and the properties of the concrete gas.

The equation of the energy along the line of the flow and the equation of the state will have the form

$$\frac{\mu}{2}(v_r^2+v_\theta^2)+2\alpha i_1+(1-\alpha)i_2+\alpha D=\mu i_{00}; \quad (5)$$

$$\frac{p}{\rho}=\frac{1+\alpha}{\mu}RT, \quad (6)$$

where μ is the molecular weight of the diatomic gas;

i_1 and i_2 are the molar enthalpies respectively of the monoatomic and diatomic gases;

and i_{00} is the enthalpy of retardation.

Let us note that when there occurs dissociation of the molecules, the oscillations are already considerably excited and the heat capacity of the molecules practically does not depend on the temperature, i. e., $i_2 = C_p 2T$ and $i_1 = C_p 1T$.

Let us introduce the new variables

$$\left.\begin{array}{l} x=\frac{r}{R_0};\quad \bar{\rho}=\frac{\rho}{\rho_\infty};\quad \bar{p}=\frac{p}{\rho_\infty v_\infty^2};\quad u=\frac{v_\theta}{v_\infty};\\ v=\frac{v_r}{v_\infty};\quad i=\frac{i}{v_\infty^2};\quad \bar{D}=\frac{D}{v_\infty^2}.\end{array}\right\} \quad (7)$$

Then the system (1) - (5) in the new variables will be written (the line over the values will be omitted by us)

$$u\frac{\partial u}{\partial\theta}+xv\frac{\partial u}{\partial x}+uv+\frac{1}{\rho}\frac{\partial p}{\partial\theta}=0; \quad (8)$$

$$u\frac{\partial v}{\partial\theta}+xv\frac{\partial v}{\partial x}-u^2+\frac{x}{\rho}\frac{\partial p}{\partial x}=0; \quad (9)$$

$$x\frac{\partial\rho v}{\partial x}+\frac{\partial\rho u}{\partial\theta}+2\rho v+\rho u\operatorname{ctg}\theta=0; \quad (10)$$

$$\frac{\mu(v^2+u^2)}{2}+\frac{2\alpha c_{p1}T}{v_\infty^2}+\frac{(1-\alpha)c_{p2}T}{v_\infty^2}+\alpha D=\mu i_{00}; \quad (11)$$

$$v\frac{\partial\alpha}{\partial x}+\frac{u}{x}\frac{\partial\alpha}{\partial\theta}=\frac{R_0}{v_\infty}\omega(T,p)=v_2\frac{R_0\omega(T_2,p_2)}{v_2}\frac{\omega(T,p)}{\omega(T_2,p_2)}\equiv v_2\chi f; \quad (12)$$

$$\frac{p}{\rho}=\frac{1+\alpha}{\mu v_\infty^2}RT. \quad (13)$$

Here by the exponent 2 there are designated the values after the shock wave.

$$\left.\begin{array}{l}\chi=\frac{R_0\omega(T_2,p_2)}{v_2};\\ f=\frac{\rho}{\rho_2}e^{+\frac{Dv_\infty^2}{RT_2}}\left[(1-\alpha)e^{-\frac{Dv_\infty^2}{RT}}-\frac{\rho}{\rho_D}\alpha^2\right].\end{array}\right\} \quad (14)$$

where χ is the parameter of the inequality with $|\chi| \ll 1$ flow practically frozen and with $|\chi| > 1$ flow close to equilibrium.

One should note that the equations (8) - (10) remain unchangeable with the substitution of x and ax, and the equation (12) does not. This points to the fact that in the equilibrium case the value for the withdrawal of the shock wave is proportional to R_0 but the unequilibrized flow does not have this similarity. For the oscillation relaxation this was noted in the reference [2].

2. Method of Solution

We will seek a solution of the system (8) - (12) in the form of series by degree Θ with coefficients depending on x [2], [3].

$$\left.\begin{aligned} u &= u_0(x)\Theta + u_1(x)\Theta^3 + \dots \\ v &= v_0(x) + v_1(x)\Theta^2 + \dots \\ p &= p_0(x) + p_1(x)\Theta^2 + \dots \\ \rho &= \rho_0(x) + \rho_1(x)\Theta^2 + \dots \\ T &= T_0(x) + T_1(x)\Theta^2 + \dots \\ \alpha &= \alpha_0(x) + \alpha_1(x)\Theta^2 + \dots \end{aligned}\right| \quad (15)$$

By substituting the expressions (15) in the equations (8) - (12), we shall get the infinite system of ordinary differential equations for the determination of u_i, v_i, p_i, and so on.

In the lowest approximation in accordance with Θ we have (the subscript 0 we will omit)

$$u^2 + xvu' + uv + \frac{2p_1}{\rho} = 0; \qquad (16)$$

$$vv' + \frac{p'}{\rho} = 0; \qquad (17)$$

$$x(\rho v)' + 2\rho(u + v) = 0; \qquad (18)$$

$$\frac{v^2}{2} + [2\alpha c_{p1} + (1 - \alpha)c_{p2}]\frac{T}{v_\infty^2} + \alpha D = \mu i_{00}; \qquad (19)$$

$$v\alpha' = v_{j}\chi f. \qquad (20)$$

With a sufficient degree of precision one can accept that close to the zero line of flow the shock wave proves to be a sphere concentric with the body [3]. Let us introduce the dimensionless radius of the shock wave $\lambda = \frac{R_0 + \delta}{R_0} = 1 + \bar{\delta} (\lambda \leqslant x \leqslant 1)$.

With $x = \lambda$ we have in accordance with [1], [3]

$$\left.\begin{aligned} \alpha(\lambda) &= 0; \quad v(\lambda) = -\frac{1}{\rho(\lambda)}\\ I(\lambda) &= 1; \quad p_1(\lambda) = -\frac{2}{\gamma_2' + 1},\\ u(\lambda) &= 1, \quad v(1) = 0, \end{aligned}\right\} \tag{21}$$

where

$$\frac{\gamma_2'}{\gamma_2' - 1} = \frac{I(T_0)}{RT_2}. \tag{22}$$

The system (16) - (20), (13) does not turn out to be closed since it contains seven unknowns. Numerical computations showed that both for the equilibrized and for the nonequilibrized flow v(x) on the zero line of the flow changes almost linearly (4). Therefore by stating v(x) in the form

$$v(x) = v(\lambda) + v'(\lambda)(x - \lambda) + \frac{1}{2} v''(\lambda)(x - \lambda)^2, \tag{23}$$

we will have a closed system of equations for the determination of v, u, p, ρ, I, α, p_1.

By making use of the system (16) - (20), (13) and the boundary conditions (21) we find $v'(\lambda)$ and $v''(\lambda)$.

$$v'(\lambda) = \left| \frac{\frac{2\mu c_{p2}}{R} p(1 - \rho) + v\chi \left[D\rho - \frac{2\mu}{R}(c_{p2} - c_{p1}) p \right]}{v\left[\mu - \frac{\mu c_{p2}}{R}(1 - p\rho)\right]} \right|_{x=\lambda}. \tag{24}$$

$$\left.\begin{aligned} v''(\lambda) = \Bigg\{ \Big[&-\mu(v')^2 - \frac{4\mu}{R}(c_{p2} - c_{p1})\chi \left[\frac{p}{\rho}\chi + \frac{2p}{\lambda}(1 + v) + \right.\\ &\left. + (p + v)v' \right] + \frac{\mu c_{p2}}{R}\left[2(v')^2 - \frac{2v'p}{\lambda}\left(\rho - 2 - \frac{1 + v}{p}\right) + \right.\\ &\left. + \frac{2p}{\lambda^2}(v + 2 - \rho + 2p_1) \right] + \chi\left[\frac{2\mu p}{\rho R}(c_{p2} - c_{p1}) - D \right] \times\\ &\times \left[2\frac{p}{\lambda}(1 + v) + pv' - \chi + \frac{D\rho^2}{\mu p^2}\left(-vv' - 2p\frac{1 + v}{\lambda} - \right.\right.\\ &\left.\left. - pv' - \frac{p}{\rho}\chi \right) \right] - \frac{v'}{v} \Bigg\} \cdot \left[\mu v - \frac{\mu c_{p2}}{R}(p + v) \right]^{-1} \Bigg|_{x=\lambda} \end{aligned}\right\} \tag{25}$$

If in the equation (23) we substitute the relationships (24), (25) and put x = 1 (v(1) = 0), then we will get an algebraic equation of the fourth degree relative to $\lambda = 1 + \bar{\delta}$. Hence it is easy to find the value for the withdrawal of the shock wave along the zero line of flow.

In continuation let us make use of the fact that the term $\frac{\mu v^2}{2}$ in the equation (23) is small in comparison with the remaining terms [5], and it can be disregarded. Besides this the pressure along the zero line changes insignificantly and it can be considered as constant and equal to $p = 1 - \frac{3}{4}\,\frac{\rho_{00}}{2}$ in accordance with [6]. Then from the equations (19), (20), (13) we shall find $\alpha = \alpha(x)$, $T = T(x)$ and $\rho = \rho(x)$. In particular for α we have the expression

$$a = \chi \frac{v(0)}{v}\,\frac{c_p bp}{RT_0\rho(0)}\, e^{\frac{bx^2}{RT_0}} \left[e^{-\frac{c_p bp}{T_0 R}} - \frac{c_{p2} bp}{RT_0\rho_D} \left(\frac{1}{1+a}\right)^2 \right]. \quad (26)$$

where

$$b = \frac{1 - a\,\frac{c_{p'} - c_{p1}}{c_{p2}}}{1 - a\,\frac{D}{\mu\omega}}. \quad (27)$$

Afterwards from the equation (17) it is possible to determine the profile of the pressure and find the second approximation for $\alpha(x)$, $\rho(x)$, $T(x)$ already with the taking into account of the corrected pressure. However, the expressions obtained prove to be quite cumbrous.

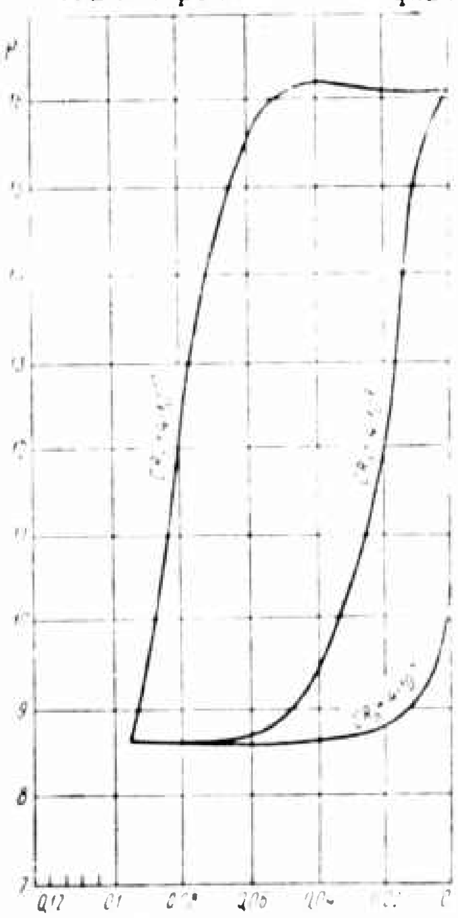

Figure 1.

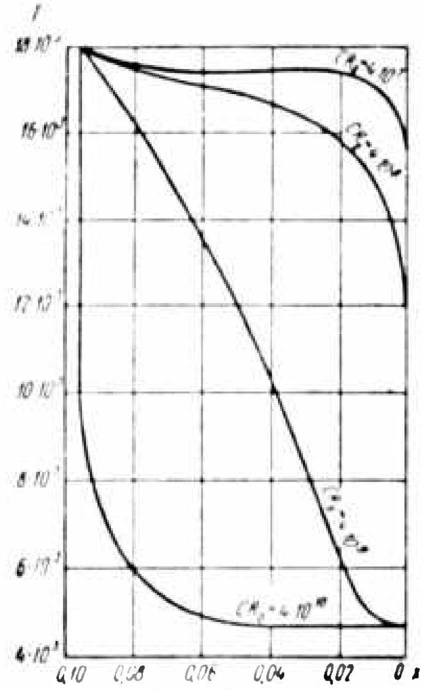

Figure 2.

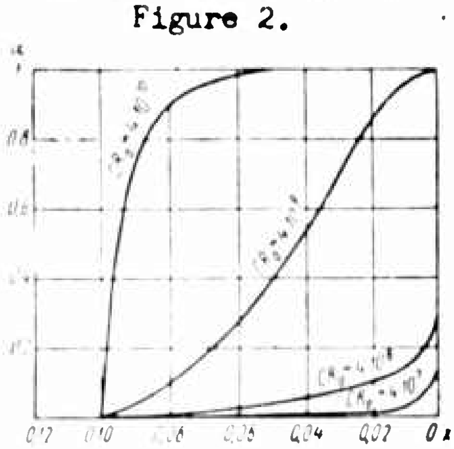

Figure 3.

3. Evaluation of the Results Obtained

A program was compiled for the BESM-2 and computations were made for a wide range of parameters of the inequality of χ and different figures for M. From the results obtained one can draw the following basic conclusions:

1. The value for the departure monotonically decreases with the increase in the M number with a given CR_o (C is the constant which characterizes the speed of the reaction, and R_o is the radius of the curvature at the critical point) and increases with the decrease in CR_o with a given M.

2. For the interval under consideration of the M numbers and the parameter of inequality χ the profiles of speed prove to be practically linear.

3. The profiles of temperature, density and degree of dissociation substantially depend on χ. With $|\chi| \ll 1$ the basic change in these values occurs close to the body, and with $|\chi > 1$ right behind the shock wave.

As an example in Figures 1, 2 and 3, respectively, profiles of density, temperature and degree of dissociation for M number 20 at $CR_o = 4 \cdot 10^7$, $|\chi| = 2.1683 \cdot 10^{-2}$, $CR_o = 4 \cdot 10^9$, $|\chi| = 0.21683$, $CR_o = 4 \cdot 10^{10}$, $|\chi| = 2.1683$ are shown.

In conclusion we consider it our duty to express our thanks to T. Ya. Timofeyev for his aid in compiling the program and carrying out the computations.

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